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Dichromacy characterized by chrominance planes

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Abstract

Dichromacy is described in terms of dichromatic opponent colour spaces. By means of the perceptual criteria 'equally bright', 'neither blue nor yellow' and 'neither red nor green' and embedding in a three-dimensional colour space, it is possible for each type of dichromat to quantify a null-chrominance plane and a null-luminance plane, both of which intersect in the missing colour. These two null planes (or the trace of their intersection with the chromaticity chart) are the chromaticities of the dichromatic opponent primaries. Since a null-luminance plane contains only colour ('chrominance'), it is simply a chrominance plane. Under the assumption that the retinal short-wavelength cones do not contribute to luminance, the chrominance planes of the three types of dichromats intersect in a common straight line, the 'blue' fundamental primary vector. This constellation may serve as a general characterisation of dichromacy. © 1998 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The three classical types of human dichromacy are protanopia, deuteranopia and tritanopia [1]. Their physiological basis is the absence in each type of one cone photopigment in the retina. An adequate description of these phenomena may be based on the concept of a fundamental colour space, also known as the cone excitation space [2]. Strictly speaking, such a description tells us nothing about which kind of colours a dichromat sees. In view of this circumstance, we would like to make use of dichromatic opponent-colour spaces, which do give us some idea of what dichromats see. As we will show, the determining elements of a dichromatic opponent-colour space are the missing colour [3], the null-luminance plane and the null-chrominance plane (for the term 'chrominance', cf. [4]). We will present these elements for the case of deuteranopia.

The basic experimental and theoretical reference system is a colour-matching space, which is assumed to be linear. In construing the opponent-colour spaces, perceptual criteria other than the strict colour match are applied. These serve solely to single out subspaces [5]; hence, they do not affect the linear structure of the colour space itself. Therefore, we do not enter the realm of 'higher colour metrics' in the sense of Schrödinger [3]. Any inadequacy in the fulfilment of the criteria (the heterochromatic luminance match, say), will only mean that the 'true' subspace has not been singled out exactly.

2. Dichromacy exemplified by deuteranopia

For measurements and for purposes of representation, we use the three-dimensional vectorial colour-matching space of Wright [6], in which the three basis vectors are realised by monochromatic radiations ('primaries') with wavelengths 460 nm (blue, B), 530 nm (green, G) and 650 nm (red, R). We denote this vector space by ${}^{3}V_{\mathbf{BGR}}$, where the superscript 3 indicates the dimension of the space and the subscript BGR denotes the additive components (an analogous notation is used for the one- and two-dimensional subspaces introduced later).

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2.1. The missing colour $\boldsymbol{D} \in M_D$

Within the colour-matching space, the deuteranopic missing colour $\boldsymbol{D}$ indicates the direction or chromaticity for which a deuteranope has no colour sensation at all. The perceptual criterion applied by the deuteranope is 'indistinguishably equal', i.e. the colour match. If the two colours $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$ look the same to a deuteranope, we can indicate this by writing

$$\boldsymbol{C}_2 \overset{\mathrm{d}}{\equiv} \boldsymbol{C}_1 \tag{1}$$

(designating the deuteranope's match of two colour stimuli by the symbol $\overset{\mathrm{d}}{\equiv}$). Because of relation Eq. (1), the difference

$$\Delta \boldsymbol{C} = \boldsymbol{C}_2 - \boldsymbol{C}_1 \tag{2}$$

represents a null vector for the deuteranope. In other words, within the two-dimensional deuteranopic vector space ${}^2\mathrm{V}_{\mathrm{BR}}$, $\Delta \boldsymbol{C} = \mathrm{O}$ ($\Delta \boldsymbol{C}$, $\mathrm{O} \in {}^2\mathrm{V}_{\mathrm{BR}}$), whereas $\Delta \boldsymbol{C}$ is not a null vector within ${}^3\mathrm{V}_{\mathrm{BGR}}$. In this space, $\Delta \boldsymbol{C}$ represents the missing colour

$$\boldsymbol{D} = \Delta \boldsymbol{C} \tag{3}$$

($\boldsymbol{D}$, $\Delta \boldsymbol{C} \in {}^3\mathrm{V}_{\mathrm{BGR}}$).

Experiment [7,8] shows that all difference vectors $\Delta \boldsymbol{C}$ have a common point in the chromaticity chart, hence, in the three-dimensional vector space, $\boldsymbol{D}$ lies in a straight line through the origin. This line is a one-dimensional subspace, which we denote by ${}^1\mathrm{M}_{\mathrm{D}}$. Strictly speaking, ${}^1\mathrm{M}_{\mathrm{D}}$ itself is the deuteranopic missing colour, since it is this subspace which vanishes under a linear mapping that reduces trichromacy to deuteranopia [5,9,10].

2.2. The null-chrominance plane ${}^2N_D$

A deuteranope can produce a binary colour mixture

$$\boldsymbol{C}_{\mathrm{N}} = u\boldsymbol{C}_3 + v\boldsymbol{C}_4 \tag{4}$$

(u, v variable; for convenience, $u + v = 1$) of two given colours $\boldsymbol{C}_3$ and $\boldsymbol{C}_4$ that conforms to the perceptual criterion 'neither blue nor yellow'. For the deuteranope, such colours $\boldsymbol{C}_{\mathrm{N}}$ are chrominance-free. Experiment [11,12,9] shows that to a very good approximation, the chromaticity loci of such colours all lie on a straight line $\mathrm{n_d}$. This means that the colour vectors $\boldsymbol{C}_{\mathrm{N}}$ lie in a plane containing the origin, i.e. in a two-dimensional subspace ${}^2\mathrm{N}_{\mathrm{D}}$, which we refer to as the null-chrominance or achrominance plane. The straight line $\mathrm{n_d}$ in which ${}^2\mathrm{N}_{\mathrm{D}}$ intersects the chromaticity chart is the so-called neutral zone.

The subspace ${}^2\mathrm{N}_{\mathrm{D}}$ vanishes under the mapping that reduces the trichromatic colour space to the deuteranopic chrominance component [11,5].

2.3. The null-luminance plane ${}^2A_D$

The counterpart of a chrominance-free colour would be a luminance-free colour. Since such colours cannot be realised by real physical stimuli, one resorts to luminance-free difference vectors: Using the criterion of minimally distinct border [13,14], the deuteranope makes a luminance match between two colours $\boldsymbol{C}_5$ and $\boldsymbol{C}_6$ of different chromaticities. The difference of these vectors is a luminance-free colour vector

$$\boldsymbol{C}_{\mathrm{A}} = \boldsymbol{C}_5 - \boldsymbol{C}_6 \tag{5}$$

To a good approximation, the chromaticity loci of such colours lie on a straight line $\mathrm{a_d}$ in the imaginary region of the chromaticity chart [8,12], which again means that the colour vectors $\boldsymbol{C}_{\mathrm{A}}$ lie in a plane containing the origin. We thus obtain another two-dimensional subspace ${}^2\mathrm{A}_{\mathrm{D}}$. Schrödinger [15,16] named this subspace the alychne (alychne = lightless; see also [17]). The straight line $\mathrm{a_d}$ in which ${}^2\mathrm{A}_{\mathrm{D}}$ intersects the chromaticity chart is the so-called alychne trace. The subspace ${}^2\mathrm{A}_{\mathrm{D}}$ vanishes under the mapping that reduces the trichromatic colour space to the deuteranopic luminance component. The imaginary colours $\boldsymbol{C}_{\mathrm{A}} \in {}^2\mathrm{A}_{\mathrm{D}}$ contain no luminance, but only chrominance. Therefore, the subspace ${}^2\mathrm{A}_{\mathrm{D}}$ may be referred to as the deuteranopic chrominance plane.

2.4. Summary of deuteranopia

The missing colour $\boldsymbol{D}$ being a general null-vector for the deuteranope, it is also a null-vector with respect to chrominance. Therefore, $\boldsymbol{D}$ must lie in the null-chrominance plane ${}^2\mathrm{N}_{\mathrm{D}}$. Similarly, $\boldsymbol{D}$ is also a null vector with respect to luminance, hence it must lie in the null-luminance plane ${}^2\mathrm{A}_{\mathrm{D}}$. Together, these two statements mean that $\boldsymbol{D}$ lies in the intersection of the subspaces ${}^2\mathrm{N}_{\mathrm{D}}$ and ${}^2\mathrm{A}_{\mathrm{D}}$. In other words, the null-chrominance plane (neutral zone) and the null-luminance plane (alychne) intersect in the missing colour, as do their loci (traces) $\mathrm{n_d}$ and $\mathrm{a_d}$ in the chromaticity chart. This is seen in the middle of Fig. 1, where the lines $\mathrm{n_d}$ and $\mathrm{a_d}$ intersect in the chromaticity point locus $\boldsymbol{D}$ in which the deuteranopic missing colour $\boldsymbol{D}$ passes through the plane of the chromaticity chart. Thus, the subspace ${}^1\mathrm{M}_{\mathrm{D}}$ is already determined by the subspaces ${}^2\mathrm{N}_{\mathrm{D}}$ and ${}^2\mathrm{A}_{\mathrm{D}}$, but of course it is clearly desirable to have an independent experimental determination of the missing colour.

We can now establish a deuteranopic opponent-colour system ${}^2\mathrm{V}_{\mathrm{KL}}$ by taking a luminance-free chrominance primary $\boldsymbol{K} \in {}^2\mathrm{A}_{\mathrm{D}}$ such that $\boldsymbol{K} \neq \boldsymbol{D}$ and a chrominance-free luminance primary $\boldsymbol{L} \in {}^2\mathrm{N}_{\mathrm{D}}$ such that $\boldsymbol{L} \neq \boldsymbol{D}$. Then, any colour C has the vector representation within the opponent space

$$\boldsymbol{C} = K\boldsymbol{K} + L\boldsymbol{L} \in {}^2V_{KL} \tag{6}$$

where K is a chrominance di-stimulus value and L is a luminance di-stimulus value. In this frame of reference, the null-chrominance plane is characterised by $K = 0$, the null-luminance plane by $L = 0$.

3. A synopsis of dichromacy

Protanopia and tritanopia may be treated in a similar fashion. Of course, the instrumental colour space $^3V_{BGR}$ that was used in deriving the relationships presented so far is indispensable when doing experiments. However, the fundamental colour space is less arbitrary. Following Judd [18], we represent the fundamental colour space by the fundamental primaries $\boldsymbol{P}$, $\boldsymbol{D}$, $\boldsymbol{T}$ and the corresponding fundamental tri-stimulus values P, D, T:

$$\boldsymbol{C} = P\boldsymbol{P} + D\boldsymbol{D} + T\boldsymbol{T} \in {}^3V_{PDT} \tag{7}$$

The designations indicate that $\boldsymbol{P}$, $\boldsymbol{D}$ and $\boldsymbol{T}$ have the chromaticities of the protanopic, deuteranopic and tritanopic missing colours, respectively. The fundamental primaries are all imaginary, i.e. their chromaticities lie outside the gamut of real colours. According to the classical hypothesis of König and Dieterici [19], P is absent in protanopes, D in deuteranopes and T in tritanopes. Hence, the protanope's fundamental colour space is $^2V_{DT}$, the deuteranope's $^2V_{PT}$ and the tritanope's $^2V_{PD}$.

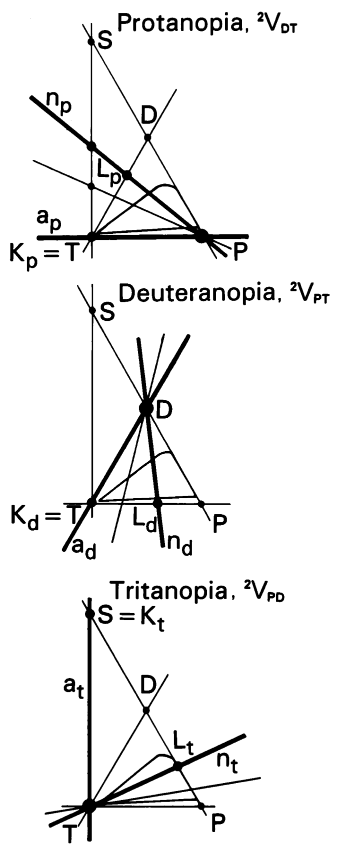

Fig. 1. Dichromatic vision. The corners of the equilateral triangle PDT are the chromaticities of the protanopic, deuteranopic, tritanopic missing colours, identified with the fundamental primaries. The rays n_p, n_d, n_t are the neutral zones, a_p, a_d, a_t are the alychne traces. K_i (i = p, d, t) are the chromaticities of the dichromatic chrominance primaries (luminance-free), L_i (i = p, d, t) are the chromaticities of the dichromatic luminance primaries (chrominance-free). The large filled circles are the vertices of the dichromatic chromaticity pencils (modified from [23]).

Fig. 1 shows three versions of the fundamental chromaticity chart, with the fundamental primaries arranged in an equilateral triangle. The fundamental chromaticity charts of the three types of dichromat have shrunk to binary mixture lines. The neutral zones n_p, n_d, n_t and the alychne traces a_p, a_d, a_t are shown as heavy lines. These lines are the traces of the crucial null-planes, which contain the origin of $^3V_{PDT}$ and are thus subspaces of $^3V_{PDT}$.

Apparent in Figs. 1 and 2 is the fact that the alychne traces (a_p, a_d, a_t) intersect in a common chromaticity locus, viz. that of the fundamental primary $\boldsymbol{T}$, which represents the retinal 'blue' cone process. Considered within $^3V_{PDT}$, the three null-luminance planes ('chrominance planes') ^2A_P, ^2A_D, ^2A_T intersect in the axis given by the fundamental primary $\boldsymbol{T}$. Hence, the axis given by $\boldsymbol{T}$ lies in these planes and all three of them intersect in this straight line. Fig. 2 provides a spatial view of this phenomenon. Physiologically, it would appear to be an exclusively postreceptoral, i.e. opponent-colour phenomenon. We consider the properties demonstrated in Fig. 2 to be suited for an intrinsic characterisation of classical dichromacy.

4. Discussion

As shown in Fig. 1, not only the neutral zones n_p, n_d and n_t are examples of dichromatic confusion lines, so too are the alychne traces a_p, a_d and a_t: they are imaginary confusion lines. Real confusion lines are determined by using the criterion 'indistinguishably equal', suggesting that a confusion line represents a dichromatic chromaticity. By definition, the loci of the chrominance primaries $\boldsymbol{K}_p$, $\boldsymbol{K}_d$ and $\boldsymbol{K}_t$, being luminance-free, lie on the alychne traces (Fig. 1).

In protanopia and deuteranopia, the chromaticity loci of the respective chrominance primaries $\boldsymbol{K}_p$ and $\boldsymbol{K}_d$ coincide with that of the 'blue' fundamental primary $\boldsymbol{T}$ (Fig. 1). This is a special feature of classical dichromacy and stems from the fact that the 'blue' retinal cones contribute little or nothing to luminance. Therefore, the alychne traces a_p, a_d and a_t (Fig. 1) intersect in a common point, the locus $\boldsymbol{T}$. That is, the three null-luminance planes (or 'chrominance planes') ^2A_P, ^2A_D and ^2A_T intersect in a common axis, the 'blue' primary vector $\boldsymbol{T}$. These features are illustrated in Fig. 2. The protanopic and deuteranopic alychne traces a_p and a_d

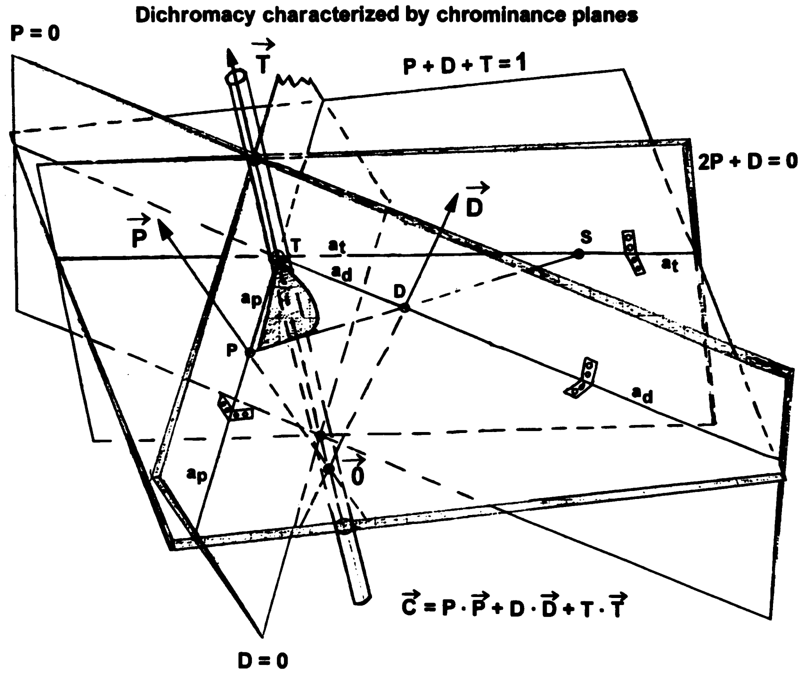

Fig. 2. Spatial view of the fundamental colour space ${}^{3}V_{PDT}$. The protanopic chrominance plane (null-luminance plane) $D = 0$, the deuteranopic chrominance plane $P = 0$ and the tritanopic chrominance plane $2P + D = 0$ intersect in a common straight line given by the 'blue' fundamental primary vector $\boldsymbol{T}$, correspondingly, the lines a_p, a_d and a_t intersect in a common point T within the fundamental chromaticity chart.

become sides and $\boldsymbol{K}_p$ and $\boldsymbol{K}_d$ become corners of the fundamental triangle. These important features are missing in tritanopia: a_t is not a side and $\boldsymbol{K}_t$ is not a corner of the fundamental triangle. One reason for this tritanopic disparity may be the greater similarity of tritanopia to normal trichromacy: the tritanopic alychne trace a_t is close to the trichromat's alychne trace and the tritanopic neutral zone n_t resembles the hue yellow of the normal trichromat.

The properties shown in Figs. 1 and 2 may reflect the pivotal role of the 'blue' cones in post-receptoral neuronal processing in the retina and hence in chromatopoesis [20], as well as their ancient role in the evolution of colour vision (cf. [21,22].

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